Comment on "Two-photon decay of the sigma meson"

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(Dated: November 16, 2008)

We comment on a recent paper by Giacosa, Gutsche, and Lyobovitskij, in which it is argued that a quarkonium interpretation of the σ meson should give rise to a much smaller two-photon decay width than commonly assumed. The reason for this claimed discrepancy is a term in the transition amplitude, necessary for gauge invariance, which allegedly is often omitted in the literature, including the work of the present authors. Here we show their claims to be incorrect by demonstrating, in the context of the Quark-Level Linear σ Model, that the recently extracted experimental value $\Gamma_{\sigma\to2\gamma} = (4.1 \pm 0.3)$ keV is compatible with a $q\bar{q}$ assignment for the σ, provided that meson loops are taken into account as well.

PACS numbers: 13.40.Hq, 14.40.Cs, 13.25.Jx, 12.39.Ki

I. INTRODUCTION

In a recent paper [1], Giacosa, Gutsche, and Lyubovitskij (GGL) studied the two-photon decay width of the σ meson, alias $f_0(600)$ [2], based on the presupposition that it is a $q\bar{q}$ state. They employed two simple perturbative sigma models, one purely local, comprising σ, π, quark and antiquark fields, and the other nonlocal, with only σ, q, and $\bar{q}$, besides an extended covariant vertex function. The principal result of their work was that, in contrast with what is generally assumed, a $q\bar{q}$ assignment for the σ should lead to a width $\Gamma_{\sigma\to\gamma\gamma}$ much smaller than the recently reported value of (4.1 ± 0.3) keV resulting from an analysis by Pennington [3], as well as the 3 values given in the 2006 PDG tables [2], and probably even less than 1 keV. Therefore, GGL concluded that, if the large experimental $\gamma\gamma$ width is confirmed, a quarkonium interpretation of the σ is not favored, *"contrary to usual belief."* As an explanation for their very small $\Gamma_{\sigma\to\gamma\gamma}$ prediction, GGL argued that a term in the quark-triangle loop diagram, necessary for gauge invariance, largely cancels the lead term, thus resulting in a small total amplitude. Moreover, GGL claimed that the former term is *"often neglected"*, including in previous work of ours and our co-authors [4, 5, 6, 7].

In this Comment, we shall show that GGL are mistaken on several points. First of all, we have *not* unduly neglected any term in the evaluation of the quark triangle diagram in Refs. [4, 5, 6, 7]. When we disregarded the term in question, this was fully justified, since the term was zero or negligible. Secondly, the small $\Gamma_{\sigma\to\gamma\gamma}$ value obtained by GGL is a consequence of a very low σ mass, in combination with a relatively large constituent quark mass, at least in the local case. For the nonlocal Lagrangian, their tiny $\Gamma_{\sigma\to\gamma\gamma}$ value is rather an indication for the inadequacy of the Lagrangian itself. Thirdly, we demonstrate, by explicit calculation, how important meson-loop contributions are, which is in principle admitted by GGL, but not concretized.

In Sec. II of this Comment, we study in detail the two-photon width of the σ meson, in the context of the quark-level linear σ model (QLLσM) [8], showing that a good agreement with data is achieved. In Sec. III we present our conclusions.

II. TWO-PHOTON WIDTH OF THE σ IN THE QLLσM

Given the scalar amplitude structure [5, 6, 9] $\mathcal{M}\epsilon_\nu(k')\epsilon_\mu(k)\,(g^{\mu\nu}k'\cdot k - k'^\mu k^\nu)$, the rate for the decay of a scalar meson S into two photons reads

$$\Gamma(S\to\gamma\gamma) = \frac{m_S^3|\mathcal{M}_{S\to\gamma\gamma}|^2}{64\pi} \ . \tag{1}$$

If one assumes, as GGL do, that the σ is a scalar $q\bar{q}$ state, then the principal contribution to the amplitude $\mathcal{M}_{\sigma\to\gamma\gamma}$ comes from the up and down quark triangle diagrams (see e.g. FIG. 1 in Ref. [1]), yielding (with $N_c = 3$)

$$\mathcal{M}^{n\bar{n}}_{\sigma\to\gamma\gamma} \;=\; \frac{5\alpha}{3\pi f_\pi}2\xi_n[2 + (1-4\xi_n)I(\xi_n)] \ , \tag{2}$$

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where $\alpha = e^2/4\pi$, $\xi_n = m_n^2/m_\sigma^2$ (n stands for u or d), and $I(\xi)$ is the triangle loop integral given by

$$I(\xi)\begin{cases} = \dfrac{\pi^2}{2} - 2\log^2\left[\sqrt{\dfrac{1}{4\xi}} + \sqrt{\dfrac{1}{4\xi} - 1}\right] + \\ \quad 2\pi i \log\left[\sqrt{\dfrac{1}{4\xi}} + \sqrt{\dfrac{1}{4\xi} - 1}\right] \;\; (\xi \le 0.25)\,, \\ = 2\arcsin^2\left[\sqrt{\dfrac{1}{4\xi}}\right] \;\; (\xi \ge 0.25)\,. \end{cases} \tag{3}$$

These Eqs. (2) and (3) exactly correspond to Eqs. (2) and (4) in Ref. [1], with the proviso that GGL defined the σ-$\bar{q}$-q coupling in their Lagrangian as $g_\sigma/\sqrt{2}$ instead of our QLLσM coupling g, the latter being related to f_π above via the Goldberger-Treiman relation $m_q = f_\pi g$ [4, 5, 6, 7]. Ignoring for the moment possible meson-loop contributions as well as an $s\bar{s}$ component in the σ, we can use Eq. (2) to calculate $\Gamma_{\sigma\to\gamma\gamma}$, for different σ and quark masses. Also, we can check what the importance is of the term involving $I(\xi)$.

However, let us first deal with the allegation by GGL that we had erroneously neglected the $I(\xi)$ term in previous work. Well, in Ref. [4] we simply worked in the, perfectly well-defined, Nambu–Jona-Lasinio (NJL) [10] limit ($m_\sigma = 2m_q$) of the QLLσM, in which the term in question vanishes identically, using quite reasonable σ and quark masses of 630 MeV and 315 MeV, respectively. The resulting $\Gamma_{\sigma\to\gamma\gamma}$, ignoring meson-loops, would then be 2.18 keV. But accounting for an estimate of the pion-loop contribution as well yielded the prediction of 3.76 keV [4], in good agreement with experiment, then and now. In Ref. [7], Eq. (101), again the NJL limit of the QLLσM was used, but now also including an estimate for the kaon loop, besides the pion loop, leading to a slightly smaller result, but still very much larger than any of GGL's predictions (also see Ref. [11]). Finally, in Refs. [5, 6] $\Gamma_{\sigma\to\gamma\gamma}$ was not even considered, thus making the critique by GGL completely void. Moreover, note that in Ref. [5] we did use the full expressions of Eqs. (2) and (3) above when necessary, namely in the case of the $f_0(1370)$ meson.

Let us now carry out a more detailed analysis of $\Gamma_{\sigma\to\gamma\gamma}$ in a QLLσM setting, employing Eqs. (2) and (3). Working beyond the chiral limit (CL), we may take the NJL value $m_\sigma = 675$ MeV for $m_n = 337.5$ MeV [12], where m_n stands for the nonstrange (up or down) quark mass. Still neglecting $n\bar{n}$-$s\bar{s}$ mixing and meson loops, this gives $\Gamma^{q\bar{q}}_{\sigma\to\gamma\gamma}$= 2.68 keV. Taking a somewhat more realistic value of $m_\sigma = 666$ MeV [12], away from the CL, the latter width gets reduced to 2.44 keV. If we now also allow for the admixture of a small $s\bar{s}$ component in the σ, with a nonstrange-strange mixing angle of, say, -10.1° [12], then we get $\Gamma^{q\bar{q}}_{\sigma\to\gamma\gamma}$= 2.49 keV, for the often used [7] QLLσM quark masses $m_n = 337.5$ MeV and $m_s = 486$ MeV. Note that this $s\bar{s}$ component, with amplitude

$$\mathcal{M}^{s\bar{s}}_{\sigma\to\gamma\gamma} = \frac{\sqrt{2}\alpha g}{3\pi m_s} 2\xi_s[2 + (1 - 4\xi_s)I(\xi_s)]\,, \tag{4}$$

contributes with a weight factor of only $\sqrt{2}\alpha m_n/3\pi f_\pi m_s$ (using the GT relation $m_n = f_\pi g$), as compared to $5\alpha/3\pi f_\pi$ from Eq. (2) in the $n\bar{n}$ case, since the charge of a strange quark is $-1/3$ [5].

Next we are going to add meson-loop contributions as well. Now, in the framework of the QLLσM, loops with charged mesons that couple to the σ include those with pions and kaons, as well as those with the scalar mesons $\kappa(800)$ and $a_0(980)$. The expression for a gauge-invariant meson-loop contribution to the two-photon amplitude mainly differs from the quark triangle in Eq. (2) because of the presence of a seagull graph (see e.g. Ref. [9], first paper), yielding a total amplitude

$$\mathcal{M}^{MM}_{\sigma\to\gamma\gamma} = -\frac{2g'\alpha}{\pi m_M^2}\left[-\frac{1}{2} + \xi I(\xi)\right]\,, \quad \xi = \frac{m_M^2}{m_\sigma^2}\,, \tag{5}$$

where the minus sign stems from the opposite statistics with respect to the quark-loop case, and g' is the cubic QLLσM meson coupling. For the meson loops pertinent to the σ, we shall need the 3-meson couplings [5, 7, 8]

$$\begin{aligned} g_{\sigma_{n\bar{n}},\pi\pi} &= \frac{\cos^2(\phi_S)m_\sigma^2 + \sin^2(\phi_S)m_{f_0(980)}^2 - m_{\pi^\pm}^2}{2f_\pi}\,, \\ g_{\sigma_{s\bar{s}},\pi\pi} &= 0\,, \\ g_{\sigma_{n\bar{n}},KK} &= \frac{\cos^2(\phi_S)m_\sigma^2 + \sin^2(\phi_S)m_{f_0(980)}^2 - m_{K^\pm}^2}{2f_K}\,, \\ g_{\sigma_{s\bar{s}},KK} &= \frac{\sin^2(\phi_S)m_\sigma^2 + \cos^2(\phi_S)m_{f_0(980)}^2 - m_{K^\pm}^2}{\sqrt{2}\,f_K}\,, \\ g_{\sigma_{n\bar{n}},\kappa\kappa} &= \frac{\cos^2(\phi_S)m_\sigma^2 + \sin^2(\phi_S)m_{f_0(980)}^2 - m_\kappa^2}{2\,(f_\pi - f_K)}\,, \\ g_{\sigma_{s\bar{s}},\kappa\kappa} &= \frac{\sin^2(\phi_S)m_\sigma^2 + \cos^2(\phi_S)m_{f_0(980)}^2 - m_\kappa^2}{\sqrt{2}\,(f_K - f_\pi)}\,, \\ g_{\sigma_{n\bar{n}},a_0a_0} &= 3g_{\sigma_{n\bar{n}},\pi\pi}\,, \\ g_{\sigma_{s\bar{s}},a_0a_0} &= 0\,, \end{aligned} \tag{6}$$

where ϕ_S is the scalar mixing angle, and $f_K = f_\pi\,(m_s/m_n + 1)/2 \approx 1.22\,f_\pi$. The cubic coupling of the physical σ meson to the three channels is then given by

$$g'_{\sigma,MM} = \cos(\phi_S)g_{\sigma_{n\bar{n}},MM} - \sin(\phi_S)g_{\sigma_{s\bar{s}},MM}\,. \tag{7}$$

Note that we neglect here small OZI-violating corrections to the QLLσM three-meson couplings, just as in previous work of ours [5]. Such contributions will be included in a forthcoming study.

Now we are in a position to do a complete calculation of $\Gamma_{\sigma\to\gamma\gamma}$, with both quark and meson loops accounted for. Note that the imaginary part of $I(\xi)$, as given by

the $\xi < 0.25$ case in Eq. (3), will be included for the pion-loop amplitude. If we choose again a scalar mixing angle of -10.1° and take $m_\kappa = 800$ MeV, we obtain a total two-gamma width

$$\Gamma^{q\bar{q}+MM}_{\sigma\to\gamma\gamma} \;=\; 3.50\ \text{keV}\ . \tag{8}$$

This rate corresponds to a total amplitude modulus $|\mathcal{M}| = 4.88\times10^{-2}\ \text{GeV}^{-1}$, which can be decomposed in terms of the partial quark- and meson-loop amplitudes

$$\begin{aligned}
\Re\text{e}\,\mathcal{M}_{n\bar{n}} &= 4.01\times10^{-2}\ \text{GeV}^{-1}\ ,\\
\Re\text{e}\,\mathcal{M}_{s\bar{s}} &= 1.09\times10^{-2}\ \text{GeV}^{-1}\ ,\\
\mathcal{M}_{\pi\pi} &= (1.19 - i1.03)\times10^{-2}\ \text{GeV}^{-1}\ ,\\
\mathcal{M}_{KK} &= -1.83\times10^{-3}\ \text{GeV}^{-1}\ ,\\
\mathcal{M}_{\kappa\kappa} &= -2.06\times10^{-3}\ \text{GeV}^{-1}\ ,\\
\mathcal{M}_{a_0a_0} &= -1.50\times10^{-3}\ \text{GeV}^{-1}\ .
\end{aligned} \tag{9}$$

Note that here the relative sign between quark and meson loops has already been included. Also observe that the kaon, κ, and $a_0(980)$ loops reduce the contribution of the pion loop, so that the net effect of the meson loops on the two-photon width is about +40%.

Taking a somewhat more negative value for the scalar mixing angle, e.g. $\phi_S = -18^\circ$ [7], only reduces the total two-phton width to 3.39 keV. This prediction as well as the former one are fully compatible with the corresponding PDG [2] data, and also not at odds with Pennington's recent result [3].

In contrast, the sensitivity of $\Gamma_{\sigma\to\gamma\gamma}$ to the σ mass is much stronger, which is obvious from Eq. (1), relating width and amplitude via m_σ cubed. This can also by seen in FIG. 2 of the paper [1] by GGL themselves, where e.g. an m_σ of 650 MeV, with $m_q = 350$ MeV, would yield a $\Gamma^{q\bar{q}}_{\sigma\to\gamma\gamma}$ of roughly 2.5 keV, in good agreement with our value of 2.44 keV above. However, by taking a very small m_σ of 440 MeV, as GGL choose to do, one obtains a much smaller $\Gamma_{\sigma\to\gamma\gamma}$, even when meson loops are included. For instance, if we assume the σ to be purely $n\bar{n}$ and take $m_q = 250$ MeV, $\Gamma_{\sigma\to\gamma\gamma}$ becomes 0.67 keV, even with the 3 meson-loop contributions included, which should be compared to GGL's value of 0.54 keV (see TABLE I of Ref. [1]) for the pure $q\bar{q}$ case. Neglecting in this scenario the term proportional to $I(\xi)$ would indeed increase our result of 0.67 keV to 1.38 keV, but this is of course an error we have not and will not make.

At this point, we also take exception at GGL's claim *"…the results for $\Gamma_{\sigma\to\gamma\gamma}$ at a fixed pole mass of $M_\sigma = 440$ MeV as favored by recent theoretical and experimental works [16,20]"*, where their reference no. 20 is our Ref. [2], i.e., the 2006 PDG Review of Particle Physics. It is simply false to state that the PDG favors a σ pole mass of 440 MeV. The truth is that the PDG listings mention "**(400–1200)−*i*(250–500) OUR ESTIMATE**", for the $f_0(600)$ T-matrix pole (i.e., S-matrix pole) as a function of $\sqrt{s}$. On the other hand, the theoretical papers referred to by GGL include the Roy-equation analysis by Caprini, Colangelo, and Leutwyler [13], which indeed found 441 MeV for the real part of the σ S-matrix pole, besides an imaginary part of 272 MeV. However, it is a common mistake to confuse the real part of the pole with the 'mass' of a broad resonance, especially when the resonance is certainly not of a pure BW type, like e.g. the σ, which is strongly distorted due to the $\pi\pi$ threshold and the Adler zero not far below [14]. Notice that, in the latter analysis, the 'mass' of the σ at which the $\pi\pi$ phase shift passes through 90° — by definition the K-matrix pole — lies at 926 MeV. This does not mean that this *is* the σ mass, but just demonstrates the difficulty of assigning *any* specific mass to a broad non-BW resonance. Anyhow, our above choice of 666 MeV, in the context of the QLLσM, is surely more reasonable than naively taking the real part of a pole that is already significantly lower than the 'world average' [2, 16] of σ poles.

To conclude this section, we note that the $Z = 0$ compositeness condition, discussed by GGL in the context of their nonlocal Lagrangian, is manifestly satisfied in the — nonperturbative and selfconsistent — QLLσM, provided $\xi = m_q^2/m_\sigma^2 \le 0.25$, with g_σ *not* depending on m_σ.

III. CONCLUSIONS

In the present Comment we have shown that GGL incorrectly referred to and criticized our previous papers on the subject. Moreover, we have demonstrated, via an explicit and detailed calculation in the context of the QLLσM, that the reported experimental values of $\Gamma_{\sigma\to\gamma\gamma}$ give quantitative support to a $q\bar{q}$ interpretation of the σ meson, provided that one uses a reasonable σ mass and also includes meson-loop contributions, besides the quark loop considered by GGL.

Finally, let us comment on the nonlocal Lagrangian employed by GGL besides the local one. Their justification was: *"However, the local approach is no longer applicable for values of M_σ close to threshold, as will be evident from the discussion of the next section."* Well, as already mentioned above, the QLLσM is a *local* renormalizable field theory, exactly satisfying the $Z = 0$ compositeness condition close to — but below — threshold, due to its nonperturbative and selfconsistent formulation [8]. This condition can be rigorously described in both the QLLσM and the NJL model, in terms of a log-divergent gap equation [15]. The latter can also be expressed via a four-dimensional ultraviolet cutoff Λ, resulting in a value $\Lambda \approx 2.3m_q$. For a nonstrange quark mass of 337.5 MeV, this gives $\Lambda \approx 750$ MeV, which is an energy scale that clearly separates the 'elementary' σ from e.g. the 'composite' ρ meson. For further details, we refer to Ref. [15].

In contrast, GGL were probably thinking in perturbative terms when going from their local σ-model Lagrangian to the nonlocal case. In view of the numerical results of the latter model, which produces even tinier values for $\Gamma_{\sigma\to\gamma\gamma}$ than their local approach, we are led to

conclude that Nature rather disfavors a nonlocal realization of chiral symmetry than a $q\bar{q}$ interpretation of the σ meson.

This work was supported by the *Fundação para a Ciência e a Tecnologia* of the *Ministério da Ciência, Tecnologia e Ensino Superior* of Portugal, under contract POCI/FP/81913/2007.

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